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# A unified theory of collective electronic excitations in a cylindrical quantum well and quantum well wire

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**Abstract.** A self-consistent energy functional perturbation theory is developed to evaluate the linear response of an electron gas in a cylindrical quantum well (CQW) and in a quantum well wire (QWW) to an external perturbation with arbitrary space and time dependence. Collective modes of the system are determined from the poles of the appropriate response function. Intersubband excitations with angular quantum numbers as well as intrasubband modes are investigated. In particular, we predict a new kind of excitation mode, referred to as the perimeter-like magnetoplasmon.

## 1. Introduction

A self-consistent energy functional perturbation theory has been widely used to study two-dimensional (2D) electron systems [1]. Experiments and theories have shown that the lower dimensionality often modifies the properties of an electron gas in three-dimensional space dramatically. Experimentally, new unexpected phenomena have been observed. Plasmons of a 2D electron gas are a case in point. The discrete plasmon spectrum of layered films has been investigated theoretically [2] and also observed by inelastic light scattering experiments [3]. The edge magnetoplasmon with angular quantum numbers for a 2D electron gas confined to a disc geometry was predicted theoretically after the edge plasmon for a 2D electron gas trapped on a surface of liquid He was observed [4, 5]. On the other hand, stimulated by the interest in the physics and technological applications of two-dimensional quantum well semiconductor structures, researchers are now beginning to fabricate and investigate quasi-one-dimensional semiconductor structures. In particular, quantum well wires (QWW) of GaAs surrounded by $Ga_{1-x}Al_xAs$ with dimensions as small as 20 nm × 10 nm in cross section have been made by Petroff *et al* [6]. The bound state and the energy spectrum of a hydrogenic donor in QWW have been discussed [7]. The result of calculations shows that, as the wire size increases an abrupt crossover from three-dimensional to one-dimensional behaviour occurs. Collective modes in a superlattice made by QWW arrays have also been studied [8, 9]. Recently, a model of a quasi-2D electron gas in the cylindrical quantum well (CQW) was suggested [10]. In the model of CQW, undoped $Ga_{1-x}Al_xAs$ with a circular cross section is surrounded by GaAs and then undoped $Ga_{1-x}Al_xAs$. The outside of such a structure is coated by Si-doped $Ga_{1-x}Al_xAs$. Finally, an undoped AlGaAs cladding layer is used to contain them so that a cylindrical quantum well (CQW) is formed. Such a CQW structure would be an exciting system to study because the effective dimensionality of the CQW could be changed by varying the radius of the core (or undoped $Ga_{1-x}Al_xAs$ in the inside of the CQW). The model of the CQW shows the transition behaviour between one and two dimensionality, just as a layered 2D electron gas which is considered as intermediate between the two- and three-dimensional configurations. For very small radius, the electron gas behaves as a quasi-one-dimensional electron gas; for a very large radius, the electron gas in the CQW behaves as 2D electron gas. For an intermediate radius, the electron gas in the CQW has very interesting properties with some remarkable features.

Now we will present a unified picture of the collective modes of an electron gas in CQW and QWW. The electron gas in CQW is confined in the cylindrical potential well, with inside radius $a_1$ and outside radius $a_2$ ($a_2 > a_1$), and the electrons are free to move in the well. For the model of QWW, the electrons are trapped in a cylindrical quantum well with radius $a_2$. In fact, the QWW model is a special case of the model of CQW.

The paper is organised as follows. In § 2, we develop the linear response of the system to an external perturbation. In § 3, we examine the collective excitations, including the intrasubband and interssubband modes in the absence of an external magnetic field. A new kind of perimeter magnetoplasmon predicted will be described in § 4. Conclusions are given in § 5.

## 2. The linear response theory

We begin with the effective-mass Hamiltonian describing an electron in the presence of the effective potential of the space-charge layer,

$$H = -(\hbar^2/2\mu)[\partial^2/\partial z^2 + \partial^2/\partial r^2 + (1/r)\partial/\partial r + (1/r^2)\partial^2/\partial\varphi^2] - (\mathrm{i}e\hbar B_0/2\mu^* c)\partial/\partial\varphi + (e^2B_0^2/8\mu^* c^2)r^2 + V_{\mathrm{eff}}(r). \quad (1)$$

Here $(r, \varphi, z)$ are cylindrical coordinates and the external magnetic field $\boldsymbol{B}_0$ is along the $z$-axis direction. The electronic wavefunctions are given by

$$|\nu\rangle = |n, m, k\rangle = \exp(\mathrm{i}kz + \mathrm{i}m\varphi)\xi_{n,m}(r) \quad (2)$$

where $\xi_{n,m}(r)$ is the eigenfunction for motion in the effective potential $V_{\mathrm{eff}}(r)$ of a cylindrical quantum well, $m$ $(=0, \pm1, \pm2, \ldots, \pm m^0)$ is the angular quantum number and $k$ is the wave vector along the $z$ direction. The eigenvalues are

$$\varepsilon_\nu = \varepsilon_{n,m}(B_0) + (\hbar^2k^2/2\mu) \quad (3)$$

where $\varepsilon_{n,m}(B_0)$ is the energy at the bottom of the $n$th subband with angular quantum number $m$.

An external perturbating potential of the form

$$\Phi^{\mathrm{ext}}(r, z, t) = \Phi^{\mathrm{ext}}(q, \Delta m, r)\exp(\mathrm{i}\omega t - \mathrm{i}qz + \mathrm{i}\Delta m\varphi) \quad (4)$$

will induce a perturbed electron density, which in turn induces perturbed Hartree and exchange-correlation potentials. The total perturbation

$$\Phi = \Phi^{\mathrm{ext}} + \Phi^{\mathrm{H}} + \Phi^{\mathrm{xc}} \quad (5)$$

is also of the form (4).

We now introduce a single-particle density matrix defined as

$$\rho_0 = \sum f_\alpha |\alpha\rangle\langle\alpha| \quad (6)$$

where $\alpha=(n, m, k)$ is a composite index defining the non-interacting single-particle state, and $f_\alpha$ is the occupation factor. In the presence of an external perturbing potential $\Phi^{\text{ext}}$, the density matrix will be modified to

$$\rho=\rho_0+\rho'. \tag{7}$$

The perturbation $\rho'$ is to be determined from the equation of motion for the density matrix. Following the Ehrenreich-Cohen self-consistent-field prescription [11], the linear response approximation leads to

$$\langle\nu|\rho'|\nu'\rangle=\{[f_0(\varepsilon_{\nu'})-f_0(\varepsilon_\nu)]/(\varepsilon_{\nu'}-\varepsilon_\nu-\hbar\omega)\}\langle\nu|H_1|\nu'\rangle \tag{8}$$

where

$$H_1=-e\Phi(q, \Delta m, r)\exp[\mathrm{i}(\omega t-qz+\Delta m\varphi)]. \tag{9}$$

The induced electron density is given by

$$\begin{aligned}\delta n(x, t)&=\mathrm{Tr}'[\rho'\delta(\boldsymbol{x}-\boldsymbol{x}')]\\&=\sum_{\nu'\nu}\{[f_0(\varepsilon_{\nu'})-f_0(\varepsilon_\nu)]/(\varepsilon_{\nu'}-\varepsilon_\nu-\hbar\omega)\}\langle\nu|H_1|\nu'\rangle\langle\nu'|\delta(\boldsymbol{x}-\boldsymbol{x}')|\nu\rangle.\end{aligned} \tag{10}$$

Assuming no overlap between electrons in different subbands, we have

$$\begin{aligned}\delta n(r, \varphi, z, t)=\Bigg(2\sum_{nn'mk}&\{[f_0(\varepsilon_{n',m-\Delta m,k+q})-f_0(\varepsilon_{n,m,k})]/(\varepsilon_{n',m-\Delta m,k+q}-\varepsilon_{n,m,k}-\hbar\omega)\}\\&\times\langle n, m|-e\Phi(q, \Delta m, r)|n', m-\Delta m\rangle\xi_{n,m}(r)\xi_{n',m-\Delta m}(r)\Bigg)\\&\times\exp[\mathrm{i}(\omega t-qz+\Delta m\varphi)].\end{aligned} \tag{11}$$

Since the induced electron density $\delta n(r, \varphi, z, t)$ is also of the form of (4), then (11) may be written as

$$\begin{aligned}\delta n(q, \Delta m, \omega, r)=\sum_{n'nm}&\Pi_{n',m-\Delta m;n,m}(q, \omega)\\&\langle n, m|-e\Phi(q, \Delta m, r)|n', m-\Delta m\rangle\xi_{n,m}(r)\xi_{n',m-\Delta m}(r)\end{aligned} \tag{12}$$

where

$$\begin{aligned}&\Pi_{n',m-\Delta m;n,m}(q, \omega)\\&\quad=(1/\pi)\int\mathrm{d}k\{[f_0(\varepsilon_{n',m-\Delta m,k+q})-f_0(\varepsilon_{n,m,k})](\varepsilon_{n',m-\Delta m,k+q}-\varepsilon_{n,m,k}-\hbar\omega)^{-1}\}\end{aligned} \tag{13}$$

is the irreducible polarisation insertion. For the case where only the $n=0$ subband is occupied in the zero magnetic field, after algebraic manipulation equations (12) and (13) become

$$\begin{aligned}\delta n(q, \Delta m, \omega, r)=\sum_{nm}&[\Pi^*_{n,m-\Delta m;0,m}(q, \omega)\\&\times\langle n, m-\Delta m|-e\Phi(q, \Delta m, r)|0, m\rangle\xi_{n,m-\Delta m}(r)\xi_{0,m}(r)\\&+\Pi^*_{0,m-\Delta m;n,m}(q, \omega)\langle 0, m-\Delta m|-e\Phi(q, \Delta m, r)|n, m\rangle\\&\times\xi_{0,m-\Delta m}(r)\xi_{n,m}(r)]\end{aligned} \tag{14}$$

where

$$\Pi^*_{n,m-\Delta m;0,m}(q,\omega)=(1/2\pi)\int \mathrm{d}k f_0(\varepsilon_{0,m,k})\{[(\varepsilon_{n,m-\Delta m}-\varepsilon_{0,m})+(\varepsilon_{k+q}-\varepsilon_k)+\hbar\omega]^{-1}$$
$$+[(\varepsilon_{n,m-\Delta m}-\varepsilon_{0,m})+(\varepsilon_{k+q}-\varepsilon_k)-\hbar\omega]^{-1}\} \quad (14a)$$

$$\Pi^*_{0,m-\Delta m;n,m}(q,\omega)=(1/2\pi)\int \mathrm{d}k f_0(\varepsilon_{0,m,k})\{[(\varepsilon_{0,m-\Delta m}-\varepsilon_{n,m})+(\varepsilon_{k+q}-\varepsilon_k)+\hbar\omega]^{-1}$$
$$+[(\varepsilon_{0,m-\Delta m}-\varepsilon_{n,m})+(\varepsilon_{k+q}-\varepsilon_k)-\hbar\omega]^{-1}\}. \quad (14b)$$

Here $\varepsilon_{n,\Delta m,k}=\varepsilon_{n,\Delta m}+(\hbar^2k^2/2\mu^*)$ and the condition $f_0(\varepsilon_n)|_{n\neq 0}=0$ has been used in the electron quantum limit. In particular, for the absence of an external magnetic field, we can take $|n,m\rangle=|n,-m\rangle$ and $\varepsilon_{n,m-\Delta m,k}=\varepsilon_{n,-(m-\Delta m)}$. Thus a straightforward calculation at zero temperature gives the real part of the irreducible polarisation insertion:

$$\mathrm{Re}\,\Pi^*_{0,m-\Delta m;n,m}(q,\omega)=(\mu^*/2\pi\hbar^2q)\ln|(\omega^2-\omega'^2_-)/(\omega^2-\omega'^2_+)| \quad (15a)$$
$$\omega'_\pm=(\hbar q/\mu^*)|[(2\mu^*/\hbar^2)(\varepsilon_f-\varepsilon_{n,m})]^{1/2}\pm\tfrac{1}{2}q+(\mu^*/\hbar^2q)(\varepsilon_{0,m-\Delta m}-\varepsilon_{n,m})]|$$

and

$$\mathrm{Re}\,\Pi^*_{n,m-\Delta m;0,m}(q,\omega)=(\mu^*/2\pi\hbar^2q)\ln|(\omega^2-\omega''^2_-)/(\omega^2-\omega''^2_+)| \quad (15b)$$
$$\omega''_\pm=(\hbar q/\mu^*)|[(2\mu^*/\hbar^2)(\varepsilon_f-\varepsilon_{n,m})]^{1/2}\pm\tfrac{1}{2}q+(\mu^*/\hbar^2q)(\varepsilon_{n,m-\Delta m}-\varepsilon_{0,m})]|.$$

It is easy to verify that when we take $\Delta m=0$ for the intrasubband modes (or $n=0$), equations (15*a*) and (15*b*) reduce as follows:

$$\mathrm{Re}\,\chi^*=(\mu^*/\pi\hbar^2q)\ln|(\omega^2-\omega_-^2)/(\omega^2-\omega_+^2)| \quad (16)$$
$$\omega_\pm=(\hbar q/\mu^*)|k_f\pm\tfrac{1}{2}q|$$

where $k_f$ denotes the Fermi wave vector. Equation (16) is exactly the result for the one-dimensional electron gas [12].

The perturbed Hartree potential can be obtained from Poisson's equation:

$$\{\partial^2/\partial r^2+(1/r)\partial/\partial r-[(\Delta m^2/r^2)+q^2]\}\Phi^{\mathrm{H}}=(4\pi e/\varepsilon_s)\delta n(q,\Delta m,\omega,r) \quad (17)$$

where $\varepsilon_s$ is the dielectric constant of the background. The solution, $\Phi^{\mathrm{H}}$, is given by

$$\Phi^{\mathrm{H}}(q,\Delta m,r)=-(e/\varepsilon_s)\int G_{\Delta m}(r,r')\delta n(q,\Delta m,\omega,r')r'\,\mathrm{d}r' \quad (18)$$

where

$$G_{\Delta m}(r,r')=4\pi K_{\Delta m}(qr)I_{\Delta m}(qr') \qquad r'\leqslant r$$
$$G_{\Delta m}(r,r')=4\pi I_{\Delta m}(qr)K_{\Delta m}(qr') \qquad r'\geqslant r.$$

Here $K_m(x)$ and $I_m(x)$ are $m$th-order modified Bessel functions. For simplicity, we introduce the following representations:

$|1\rangle=|0,m-\Delta m\rangle \qquad |2\rangle=|0,m\rangle \qquad |3\rangle=|n,m-\Delta m\rangle \qquad |4\rangle=|n,m\rangle.$

Substituting (14) into (18), the matrix elements of $\Phi^{\mathrm{H}}$ are

$$\langle 3|[-e\Phi^{\mathrm{H}}(q,\Delta m,r)]|2\rangle$$
$$=\sum_{n'm'}[\Pi^*_{n',m'-\Delta m;0,m'}(q,\omega)A_{3,2;3',2'}(q)\langle 3'|-e\Phi(q,\Delta m,r)|2'\rangle$$
$$+\Pi^*_{0,m'-\Delta m;n',m'}(q,\omega)A_{3,2,1',4'}(q)\langle 1'|-e\Phi(q,\Delta m,r)|4'\rangle] \quad (19a)$$

$$\langle 1|[-e\Phi^{H}(q,\Delta m,r)]|4\rangle$$
$$= \sum_{n'm'} [\Pi^{*}_{n',m'-\Delta m;0,m'}(q,\omega)A_{1,4,3',2'}(q)\langle 3'|-e\Phi(q,\Delta m,r)|2'\rangle$$
$$+\Pi^{*}_{0,m'-\Delta m;n'm'}(q,\omega)A_{1,4;1',4'}(q)\langle 1'|-e\Phi(q,\Delta m,r)|4'\rangle] \quad (19b)$$

where

$$A_{3,2;3',2'} = (e^2/\varepsilon_s)\int rr'\,\mathrm{d}r\,\mathrm{d}r'\,G_{\Delta m}(r,r')\xi_{n,m-\Delta m}(r)\xi_{n',m'-\Delta m}(r')\xi_{0,m'}(r)\xi_{0,m}(r')$$
$$A_{3,2;1',4'} = (e^2/\varepsilon_s)\int rr'\,\mathrm{d}r\,\mathrm{d}r'\,G_{\Delta m}(r,r')\xi_{n,m-\Delta m}(r)\xi_{0,m'-\Delta m}(r')\xi_{n',m'}(r)\xi_{0,m}(r')$$
$$A_{1,4;3',2'} = (e^2/\varepsilon_s)\int rr'\,\mathrm{d}r\,\mathrm{d}r'\,G_{\Delta m}(r,r')\xi_{0,m-\Delta m}(r)\xi_{n',m'-\Delta m}(r')\xi_{0,m'}(r)\xi_{n,m}(r')$$
$$A_{1,4;1',4'} = (e^2/\varepsilon_s)\int rr'\,\mathrm{d}r\,\mathrm{d}r'\,G_{\Delta m}(r,r')\xi_{0,m-\Delta m}(r)\xi_{0,m'-\Delta m}(r')\xi_{n,m'}(r)\xi_{n,m}(r'). \quad (20)$$

The exchange correlation potential $\Phi^{xc}(n(r))$ is taken to be a local function of the density by the treatment of Kohn and Sham:

$$\Phi^{xc}(r,\varphi,z,t) = (\delta\Phi^{xc}[n]/\delta n)\delta n(r,\varphi,z,t). \quad (21)$$

The matrix elements of $\Phi^{xc}$ are

$$\langle 3|[-e\Phi^{xc}(q,\Delta m,r)]|2\rangle$$
$$= \sum_{n'm'} [\Pi^{*}_{n',m'-\Delta m;0,m'}(q,\omega)B_{3,2;3',2'}(q)\langle 3'|-e\Phi(q,\Delta m,r)|2'\rangle$$
$$+\Pi^{*}_{0,m'-\Delta m;n',m'}(q,\omega)B_{3,2;1',4'}(q)\langle 1'|-e\Phi(q,\Delta m,r)|4'\rangle] \quad (22a)$$

$$\langle 1|[-e\Phi^{xc}(q,\Delta m,r)]|4\rangle$$
$$= \sum_{n'm'} [\Pi^{*}_{n',m'-\Delta m;0,m'}(q,\omega)B_{1,4;3',2'}(q)\langle 3'|-e\Phi(q,\Delta m,r)|2'\rangle$$
$$+\Pi^{*}_{0,m'-\Delta m;n',m'}(q,\omega)B_{1,4;1',4'}(q)\langle 1'|-e\Phi(q,\Delta m,r)|4'\rangle] \quad (22b)$$

where

$$B_{3,2;3',2'} = -e\int r\,\mathrm{d}r(\delta\Phi^{xc}[n]/\delta n)\xi_{n,m-\Delta m}(r)\xi_{n',m'-\Delta m}(r)\xi_{0,m'}(r)\xi_{0,m}(r)$$
$$B_{3,2;1',4'} = -e\int r\,\mathrm{d}r\,(\delta\Phi^{xc}[n]/\delta n)\xi_{n,m-\Delta m}(r)\xi_{0,m'-\Delta m}(r)\xi_{n',m'}(r)\xi_{0,m}(r)$$
$$B_{1,4;3',2'} = -e\int r\,\mathrm{d}r\,(\delta\Phi^{xc}[n]/\delta n)\xi_{0,m-\Delta m}(r)\xi_{n',m'-\Delta m}(r)\xi_{0,m'}(r)\xi_{n,m}(r)$$
$$B_{1,4;1',4'} = -e\int r\,\mathrm{d}r(\delta\Phi^{xc}[n]/\delta n)\xi_{0,m-\Delta m}(r)\xi_{0,m'-\Delta m}(r)\xi_{n,m'}(r)\xi_{n,m}(r). \quad (23)$$

Putting (19), (20), (22) and (23) in (5), we have

$$\sum_{n'}\left(\sum_{m'}[M_{32;3'2'}(q,\omega)\langle 3'|\Phi(q,\Delta m,r)|2'\rangle - N_{32;1'4'}(q,\omega)\langle 1'|\Phi(q,\Delta m,r)|4'\rangle]\right)$$
$$= \langle 3|\Phi^{0}(q,\Delta m,r)|2\rangle \quad (24a)$$

$$\sum_{n'}\left(\sum_{m'}[M_{14;1'4'}(q,\omega)\langle 1'|\Phi(q,\Delta m,r)|4'\rangle - N_{14;3'2'}(q,\omega)\langle 3'|\Phi(q,\Delta m,r)|2'\rangle]\right)$$

$$=\langle 1|\Phi^0(q,\Delta m,r)|4\rangle \tag{24b}$$

where

$$M_{\alpha\beta;\alpha'\beta'}(q,\omega)=\delta_{\alpha,\alpha'}\delta_{\beta\beta'}-\Pi^*_{\alpha'\beta'}(q,\omega)(A_{\alpha\beta;\alpha'\beta'}-B_{\alpha\beta;\alpha'\beta'}) \qquad \alpha,\beta=3,2 \text{ or } 1,4 \tag{25a}$$

$$N_{4-\alpha 6-\beta\alpha'\beta'}(q,\omega)=\Pi^*_{\alpha'\beta'}(q,\omega)(A_{4-\alpha 6-\beta;\alpha'\beta'}-B_{4-\alpha 6-\beta;\alpha'\beta'}) \qquad \alpha,\beta=3,2 \text{ or } 1,4. \tag{25b}$$

Since $M_{\alpha\beta;\alpha'\beta'}(q,\omega)$ and $N_{4-\alpha 6-\beta;\alpha'\beta'}$ represent the matrix of the dielectric function, clearly the response of the system to the external perturbation depends on the inverse of the matrices $M$ and $N$.

## 3. Intrasubband and intersubband modes

In this section, we discuss the excitation modes in the absence of the static magnetic field. The condition for the collective modes of the system is that self-sustaining oscillations in the electron density occur. This means that $\Phi^{\text{ext}}=0$, while $\Phi\neq 0$. If the $n=0$ and $n=1$ subbands are included, a set of equations (24*a*) and (24*b*) determining the dispersion relation of collective modes becomes

$$\sum_{n'm'}\left(\sum[M_{32;3'2'}(q,\omega)\langle 3'|\Phi(q,\Delta m,r)|2'\rangle - N_{32;1'4'}(q,\omega)\langle 1'|\Phi(q,\Delta m,r)|4'\rangle]\right)=0 \tag{26a}$$

$$\sum_{n'}\left(\sum_{m'}[M_{14;1'4'}(q,\omega)\langle 1'|\Phi(q,\Delta m,r)|4'\rangle - N_{14;3'2'}(q,\omega)\langle 3'|\Phi(q,\Delta m,r)|2'\rangle]\right)=0. \tag{26b}$$

For intrasubband excitations, only the $n=n'=0$ element contributes. In this case, following Das Sarma and Madhukar [13], we consider the approximation where the electron density profile in the radial direction is a $\delta$ function, $|\xi_{n,m}(r)|^2=\delta(r-a)/r$, where $a_1<a<a_2$. The non-interacting single-particle energy for the electron with effectiveness mass $\mu^*$ in CQW can be written as

$$\varepsilon_{n,m}=\varepsilon_n+(\hbar^2/2\mu^*)(m^2/a^2) \qquad m=0,\pm 1,\pm 2,\ldots,\pm m^0. \tag{27}$$

In this case, $|1\rangle=|3\rangle$, $|2\rangle=|4\rangle$ and $\Pi^*_{14}=\Pi^*_{32}$. Therefore after some algebraic manipulation, equations (26*a*) and (26*b*) reduce as follows:

$$1-\chi^*_{0;0}(q,\Delta m,\omega)[A(q,\Delta m)-B(q,\Delta m)]=0 \qquad \Delta m=0,\pm 1,\ldots,\pm m^0 \tag{28}$$

where the real and imaginary parts of the polarisability at absolute zero temperature become, respectively,

$$\text{Re}\,\chi_{0;0}(q,\Delta m,\omega)=(\mu^*/\pi\hbar^2 q)\sum \ln|(\omega^2-\omega_-^2)/(\omega^2-\omega_+^2)| \tag{29}$$

$$\text{Im}\,\chi_{0;0}(q,\Delta m,\omega)=-(\mu^*/\hbar^2 q)\sum\{H[-|k_f N(m)-\mu^*\omega/(\hbar q)|+k_z(m)] - H[-|k_f N(m)+\mu^*\omega/(\hbar q)|+k_z(m)]\} \tag{30}$$

where

$$k_z = (k_f^2 - m^2/a^2)^{1/2} \qquad m^0 = [k_f a]$$
$$N(m) = q/(2k_f) + m\Delta m/(qk_f a^2) + \Delta m^2/(2qk_f a^2)$$
$$\omega_\pm = (\hbar q/\mu^*)|(k_f^2 - m^2/a^2)^{1/2} \pm [\tfrac{1}{2}q + m\Delta m/(qa^2) + \Delta m^2/(2qa^2)]|.$$

Here $[k_f a]$ denote integral parts of $k_f a$. It is worthwhile mentioning that for a given electron gas density $n_0$ the maximum angular quantum number $m^0$ is directly related to the radius because the ranges for integral and summation to (14) are determined by the Fermi energy $E_f$. $k_f$ is the Fermi wave vector which is determined by

$$n_0 = (1/\pi^2 a^2)\{k_f a + 2[(k_f a)^2 - 1]^{1/2} + \ldots + 2[(k_f a)^2 - [k_f a]^2]^{1/2}\}. \qquad (31)$$

Obviously the maximum angular quantum number $m^0$ is related to radius $a$. For example, for $n_0 = 10^{12}\ \mathrm{cm}^{-2}$, if we take radius $a = 100$ Å then $m^0 = 2$; if $a = 1000$ Å then $m^0 = 25$. When the radius $a < r_0 = 31$ Å, for the system with an electron gas density $n_0 = 10^{12}\ \mathrm{cm}^{-2}$, the system reduces to the one-dimensional electron gas. Therefore, we refer to $r_0$ as the critical radius. In general, the relation between the critical radius $r_0$ and the electron gas density $n$ is described by $r_0 = 1/\pi n_0^{1/2}$.

The dependence of the plasmon frequency $\omega$ on the wave vector $q$ and angular quantum number $m$ is found from (28) and (29). Choosing parameters of sample a = 100 Å, $n_0 = 10^{12}\ \mathrm{cm}^{-2}$, effective mass $\mu^* = 0.067 m_e$ and dielectric constant $\varepsilon = 12.5$, we find that equation (38) reduces to an equation of fifth power in $\omega^2$. For a given value of $\Delta m$ there are three frequency intervals $\omega_- < \omega < \omega_+$ where single-particle excitations may occur. This means that the single-particle continuum is split into three discrete continua within the region of small wave vector $q$. The collective excitations only exist outside the frequency intervals; in other words, there are three dispersion spectra of plasmons lying above $\omega_+$ for small wave vector $q$. All of them correspond to a given angular quantum number $\Delta m$. Because of the absence of magnetic field, the symmetry with respect to the rotated axis causes two-fold degeneracy for $\Delta m = \pm 1$ and $\pm 2$, respectively.

The intersubband modes for the $n = 0$ and $n = 1$ subbands can be obtained from (26). In addition to the intrasubband modes, a set of equations for intersubband modes is given by

$$\sum_{m'} [M_{32;3'2'}\langle 3'|-e\Phi|2'\rangle - N_{32;1'4'}\langle 1'|-e\Phi|4'\rangle] = 0 \qquad (32)$$

$$\sum_{m'} [-N_{14;3'2'}\langle 3'|-e\Phi|2'\rangle + M_{14;1'4'}\langle 1'|-e\Phi|4'\rangle] = 0 \qquad (33)$$

where $|3\rangle = |1, m - \Delta m\rangle$, $|4\rangle = |1, m\rangle$ and $\Delta m = 0, \pm 1, \ldots, \pm m^0$. Equations (32) and (33) are available for both the QWW and CQW systems, the difference between them lying in the choice of the different eigenfunctions $\xi_{n,m}(r)$ and eigenvalues $\varepsilon_{n,m}(k)$ which affect $M_{\alpha\beta;\alpha'\beta'}(q, \omega)$ and $N_{4-\alpha 6-\beta;\alpha'\beta'}$. If we take $m^0 = 2$ as in the previous analysis, there are also three discrete plasmon spectrum branches regardless of whether the QWW or CQW system is used in the absence of an external magnetic field. The numerical solution has not been examined, but it presents no fundamental difficulty.

## 4. Discrete perimeter magnetoplasmon

Now we consider a static external magnetic field $\boldsymbol{B}$ oriented along the $z$ direction parallel to the electron-gas layer in CQW. Obviously, the magnetic field does not affect the motion of the electrons in the $z$ direction, but there are significant influences on the circular and radial motions so that the problem becomes rather complicated. For simplicity, we take $n = n' = 0$, i.e. intrasubband modes. In the case of $\varepsilon_{0,-(m-\Delta m)} \neq \varepsilon_{0,m-\Delta m}$ and $|0, m\rangle \neq |0, -m\rangle$, equation (14) is rewritten as

$$\begin{aligned}\delta n(q, \Delta m, \omega, r) = \sum_m [&\Pi^*_{0,m-\Delta m;0,m}(q, \omega) \\ &\times \langle 0, m| - e\Phi(q, \Delta m, r)|0, m - \Delta m\rangle \xi_{0,m}(r)\xi_{0,m-\Delta m}(r) \\ &+ \Pi^*_{0,-m;0,-(m-\Delta m)}(q, \omega) \\ &\times \langle 0, -(m-\Delta m)| - e\Phi(q, \Delta m, r)|0, -m\rangle \xi_{0,-(m-\Delta m)}(r)\xi_{0,-m}(r)]\end{aligned} \tag{34}$$

where

$$\Pi^*_{0,m-\Delta m;0,m} = (1/2\pi) \int \mathrm{d}k f_0(\varepsilon_{0,m,k})/[(\varepsilon_{0,m-\Delta m} - \varepsilon_{0,m}) + (\varepsilon_{k+q} - \varepsilon_k) - \hbar\omega] \tag{34a}$$

$$\Pi^*_{0,-(m-\Delta m);0,-m} = (1/2\pi) \int \mathrm{d}k f_0(\varepsilon_{0,-m,k})/[(\varepsilon_{0,-(m-\Delta m)} - \varepsilon_{0,-m}) + (\varepsilon_{k+q} - \varepsilon_k) + \hbar\omega]. \tag{34b}$$

Similar to the previous procedure, a set of equations for intersubband modes is given by

$$\sum_{m'} (M_{12;1'2'}\langle 1'|-e\Phi|2'\rangle - N_{12;-2'-1'}\langle -2'|-e\Phi| -1'\rangle) = 0 \tag{35a}$$

$$\sum_{m'} (-N_{-2-1;1'2'}\langle 1'|-e\Phi|2'\rangle + M_{-2-1;-2'-1'}\langle -2'|-e\Phi| -1'\rangle) = 0 \tag{35b}$$

where

$$M_{\alpha\beta;\alpha'\beta'}(q, \omega) = \delta_{\alpha,\alpha'}\delta_{\beta\beta'} - \Pi^*_{\alpha'\beta'}(q, \omega)(A_{\alpha\beta;\alpha'\beta'} - B_{\alpha\beta;\alpha'\beta'}) \qquad \alpha, \beta = 1, 2 \text{ or } -2, -1 \tag{35c}$$

$$N_{-\beta-\alpha;\alpha'\beta'}(q, \omega) = \Pi^*_{\alpha'\beta'}(q, \omega)(A_{-\beta-\alpha;\alpha'\beta'} - B_{-\beta-\alpha;\alpha'\beta'}) \qquad \alpha, \beta = 1, 2 \text{ or } -2, -1. \tag{35d}$$

Here the forms of $M_{-\alpha-\beta;\alpha'\beta'}(q, \omega)$ and $N_{-\beta-\alpha;\alpha'\beta'}(q, \omega)$ are similar to (25$a$) and (25$b$), and $|-1\rangle = |0, -(m - \Delta m)\rangle$, $|-2\rangle = |0, -m\rangle$. For intrasubband excitations, we assume that the electron density profile in the radial direction is a $\delta$ function, $|\xi_{0,m}(r)|^2 = |\xi_{0,-m}(r)|^2 = \delta(r-a)/r$, where $a_1 < a < a_2$. The one-electron energy levels are given by

$$\varepsilon_{0,m,k} = (\hbar^2/2\mu^*)[k^2 + m^2/a^2 + (eB_0/c\hbar)m + (e^2a^2B_0^2/4c^2\hbar^2)] \tag{36}$$

therefore (35) reduces as follows:

$$1 - \chi^*_{0;0}(q, \Delta m, B_0, \omega)[A(q, \Delta m) - B(q, \Delta m)] = 0 \qquad \Delta m = 0, \pm 1, \ldots, \pm m^0. \tag{37}$$

The real and imaginary parts of the polarisability at absolute zero temperature become

$$\operatorname{Re} \chi_{0;0}(q, \Delta m, \omega, B_0) = (\mu^*/\pi\hbar^2 q) \sum_{m=m^0_-}^{m^0_+} \ln|(\omega^2 - \omega_-^2)/(\omega^2 - \omega_+^2)| \tag{38a}$$

$$\begin{aligned}\operatorname{Im} \chi_{0;0}(q, \Delta m, \omega, B_0) = -(\mu^*/\hbar^2 q) \sum_{m=m^0_-}^{m^0_+} \{&H[-|k_f N(m) - \mu^*\omega^2/(\hbar q)| + k_z(m)] \\ &- H[-|k_f N(m) + \mu^*\omega^2/(\hbar q)| + k_z(m)]\}\end{aligned} \tag{38b}$$

$$\omega_\pm = (\hbar q/\mu^*)|k_z(m) \pm [\tfrac{1}{2}q + \Delta m(m + a^2/2L_c^2)/(qa^2) + \Delta m^2/(2qa^2)]| \tag{38c}$$

where $L_c = (c\hbar/eB_0)^{1/2}$ is the Landau radius, $H(x)$ is the step function,

$$k_z(m) = [k_f^2 - (m/a + a/2L_c^2)^2]^{1/2}$$

$$m_+^0 = [k_f a - a^2/2L_c^2] \qquad m_-^0 = -[k_f a + a^2/2L_c^2]$$

$$N(m) = q/(2k_f) + (m + a^2/2L_c^2)\Delta m/(qk_f a^2) + \Delta m^2/(2qk_f a^2).$$

Comparing (37) and (38) with (28) and (29), we find that the difference between the presence and the absence of the external magnetic field lies in $\omega_\pm$ and the region of summation about $m$. Now $m$ is replaced by $m + a^2/2L_c^2$ and the region of summation is from $-[k_f a + a^2/2L_c^2]$ to $[k_f a - a^2/2L_c^2]$ in the case of the presence of a static magnetic field. The dependence of the plasmon frequency $\omega$ on wave vector $q$ and angular quantum number $\Delta m$ is found from (37) and (38). It is notable that (37) and (38) are invariant under the separate transformations $m$ and $\Delta m$ to $-m$ and $-\Delta m$, $B_0$ to $-B_0$, so that the basic dynamical equations do not distinguish right and left helicity. However, a definite magnetic field will split the normal modes; for example, for given parameters as before, three branches of the spectrum become six plasmon lines. Moreover, there may be a possibility that the magnetic field can excite a new magnetoplasmon without Landau damping. In particular, the magnetic field dependence of frequencies for the mode with angular quantum number $\Delta m = \pm 1$ will be interesting. When the magnetic flux closed by a cylindrical surface is just an integral multiple, $a^2/2L_c^2$ becomes an integral number so that (37) and (38) happen to be of the same form as the equations for the absence of an external magnetic field. It seems to be understood that electronic collective modes can not feel the quantised magnetic field, as well as the 'extended' state of the electrons in the quantum Hall effect. The frequency of collective excitations is dramatically varied with increasing magnetic field $B_0$, and all electronic excitation modes of this system are periodic with respect to an external magnetic field in contrast to the bulk resonance modes of 2DEG whose squared frequency increases linearly with the squared cyclotron frequency. This remarkable feature is also different from that of the edge plasmon found in experiments on an electron gas trapped on the surface of liquid He [4, 5]. The modes are new and reasonable magnetoplasmons, and we suggest that they be referred to as 'discrete perimeter modes'.

## 5. Conclusions

In this paper we have presented a unified theory of the electronic collective modes in a cylindrical quantum well (CQW) and quantum well wire (QWW) including the effects of magnetic field. We have found new kinds of quasi-1D discrete modes with angular quantum numbers: intrasubband plasmons, intersubband modes and discrete perimeter magnetoplasmons. We have shown that the intrasubband modes display the appropriate crossover behaviour (from 1D to 2D) on going from small radius to large radius. The anomalous modes, called 'discrete perimeter plasmons', would be interesting due to their peculiarities. These electronic modes can be detected by light scattering, inelastic-electron scattering and infrared-absorption measurements. In our work, we have not examined intersubband modes, but we believe that these modes also display the same pecularities as intrasubband modes. In particular, calculations of intersubband modes in the presence of a static magnetic field will be more complicated; however, some new and unexpected modes may appear. We expect that the CQW system could be made using molecular-beam epitaxy and photolithography; this is in the range of several hundred nanometres for the CQW and should not be difficult to fabricate, as shown in the experiment of [6].

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